

# Extending Cooper's theorem to $\Delta^1_3$ Turing degrees

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## Abstract

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<http://dx.doi.org/10.3233/COM-180085>

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## Keywords

arithmetical hierarchy, Ershov's hierarchy, fine hierarchy, Turing degrees,  $\Delta^1_2$ -sets

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